

A note re some geometry of cosmic ray muons

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Introduction

Some geometry re cosmic muons on our LArTPC.

Jostlein's sum rule for the total track length in a volume

Imagine a bounded object (such as a sphere or a cylinder or a cube or a shapeless blob) fully illuminated by a uniform beam of equispaced parallel rays. Then the total path length, L , of the rays within the object is independent of the angle at which the object is illuminated.

The proof is simple. Consider each ray to be represented by a square cross section rod such that the sides of the rods just touch each other to define the beam. The volume illuminated is just the total length of rods (rays) within the object multiplied by the cross section area of a rod. Since the volume of the object is independent of the orientation of the object and the cross section of each rod is fixed for a given beam, the total path length = volume (fixed)/cross section (fixed), is independent of the object orientation.

Rates on a horizontal plane:

The angular intensity of cosmic ray muons is $dN/d\Omega = R \cos^2(\theta)/m^2/s$, assumed uniform in ϕ . The quantity R is ≈ 70 for muons above 1 GeV/c and ≈ 80 for muons above 0.5 GeV/c. The number of rays, N , crossing a horizontal plane per unit area per unit time is

$$N = R \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \sin(\theta) \cos^2(\theta) \cos(\theta)$$

the last $\cos(\theta)$ is the projection of the area onto the θ direction. Writing $d\theta \sin(\theta)$ as $-d\cos(\theta)$, the integral is

$$\begin{aligned} R \times 2\pi \times \int_0^1 d\cos(\theta) \cos^3(\theta) \\ = R\pi/2 \end{aligned}$$

Rates on a vertical plane:

The number of rays, V , crossing a vertical plane per unit area per unit time is

$$V = R \times 2 \int_{-\pi/2}^{\pi/2} d\phi \cos(\phi) \int_0^{\pi/2} d\theta \sin(\theta) \cos^2(\theta) \sin(\theta)$$

where the last term ($\sin(\theta)$) and the first term ($\cos(\phi)$) are, again, the projections of the plane on the direction of the rays and the factor of 2 is because rays cross the plane from both sides. (Note the ϕ integral goes from $-\pi/2$ to $\pi/2$ only) The term $R \times 2 \int_{-\pi/2}^{\pi/2} d\phi \cos(\phi)$ give $4R$ (cf $2\pi R$ in the case of the horizontal plane). The remaining integral is

$$\int_0^{\pi/2} d\theta \sin^2(\theta) \cos^2(\theta) = 1/4 \times \int_0^{\pi/2} d\theta \sin^2(2\theta) = 1/8 \times \int_0^{\pi} dq \sin^2(q) = \pi/16$$

giving $V = R \pi/4$, one half of the flux through a horizontal plane. Note, this is the total number through the plane from both sides. Going through from one side (as is the case for entering a detector) the flux is $R \pi/8$ per unit vertical area, one quarter the flux crossing a unit horizontal area.

For a cylinder of height, h , and diameter, d , the flux of cosmic muons entering through the top, $R \pi^2 \times d^2/8$ and the flux entering through the side, $R \pi^2 \times d \times h/8$, are the same when $h = d$. (The area of the side being 4 times the area of the top.) The total in this case is $R \pi^2 \times d^2/4$. Taking muons above 0.5 GeV/c and a detector with $d = h = 35.5$ m, the flux is $80 \times \pi^2 \times 35.5^2/4 = 249,000$ per second.